Matrices & Determinants

Matrices

A matrix is an array (eg of numbers) having rows and columns. Denoted by say A_{nm} , a matrix having n rows and m columns. Elements are denoted by a_{ij} which is the element in the i^{th} row and j^{th} column.

Matrix operations:

***Addition, subtraction** are only defined for matrices of the same dimensions and is simply addition or subtraction of corresponding elements.*

So $A + B = C$ *where for all i and j* $c_{ij} = a_{ij} + b_{ij}$

***Products** of matrices are defined for matrices that are **conformable** ie if A is an n x m matrix it can be post multiplied by B if B is an m x p matrix. The resulting matrix C is an n x p matrix.*

Elements of C are $c_{ij} = \sum_{k=1}^{j} a_{ik} b_{kj}$

In general $A \times B \neq B \times A$ ie operation is not commutative. Both operations would only be defined if A and B were both square matrices (ie number of rows = number of columns)

***Inverse** of a matrix A written A^{-1} is only defined for <u>square matrices</u> and is defined by:*

$A \times A^{-1} = I$ *where I is the unit or identity matrix – diagonal elements = 1, off diagonal elements =0.*

2D Matrices and Determinants

Denoting the elements of a 2 x 2 matrix A by a,b,c,d

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *, and the determinant of det A is $ad - bc$*

then the inverse of $A = A^{-1} = (1/det\ A)\, x \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Note that things get a lot more complicated for higher order matrices.

Then we have $A \times A^{-1} = A^{-1} \times A = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

For conformable matrices any any dimension,

$AI = IA = A$ $(A^{-1})^{-1} = A$ $det(AB) = detA detB = det(BA)$

A is SINGULAR if it has no inverse (detA = 0). Determinants and inverses are only defined for square (n x n) matrices.

$(AB)^{-1} = B^{-1}A^{-1}$,

$(AB)C = A(BC)$, $A(B+C) = AB + AC$, $(A+B)C = AC + BC$

(product is associative and distributive over addition)